Math 1060–003, Fall 2013
Instructor: Kyle Steffen
October 25, 2013

Name: _____

uNID: _____

Total points: _____/55

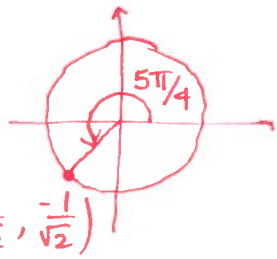
Midterm Exam #2

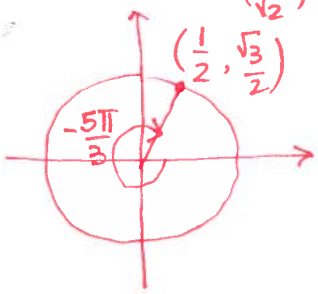
INSTRUCTIONS: You have 50 minutes to complete this exam. No calculators, notes, books, neighbors, etc. are allowed. **To receive full credit**, you must

- **Clearly show your work,**
- **Use correct units** (when applicable), and
- **Place your final answer in the rectangle** provided (when applicable).

You should *simplify all fractions and expressions* as much as possible, but you do not need to simplify square roots or rationalize denominators.

1. *(2 points)* Evaluate the following expressions.
   (*a*) $\sin \frac{5\pi}{4}$
   (*b*) $\tan\left(-\frac{5\pi}{3}\right)$





$\longrightarrow \tan\left(-\frac{5\pi}{3}\right) = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

(*a*) $-1/\sqrt{2}$

(*b*) $\sqrt{3}$

2. *(2 points)* Evaluate the following expressions.
   (*a*) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $\longrightarrow \cos\theta = -\frac{\sqrt{3}}{2}$ & $0 \le \theta \le \pi \longrightarrow \theta = 5\pi/6$
   (*b*) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ $\longrightarrow \tan\theta = -1/\sqrt{3}$ & $-\frac{\pi}{2} < \theta < \frac{\pi}{2} \longrightarrow \theta = -\pi/6$

Domain of $\cos^{-1} = [-1, 1]$
Range " " $= [0, \pi]$

Domain of $\tan^{-1} = \mathbb{R}$
Range " " $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(*a*) $5\pi/6$

(*b*) $-\pi/6$

3. *(4 points)* Evaluate the following expressions.
   (*a*) $\sin(\arctan\sqrt{3})$
   (*b*) $\arccos(\tan\pi)$

(a) $\arctan\sqrt{3} = \theta$ such that $\frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}/2}{1/2}$
(and $-\pi/2 < \theta < \pi/2$)
$\rightarrow \sin(\arctan\sqrt{3}) = \sin\theta = \sqrt{3}/2$

(b) $\arccos(\tan\pi)$
$= \arccos(0)$
$= \pi/2$

(*a*) $\sqrt{3}/2$
(*b*) $\pi/2$

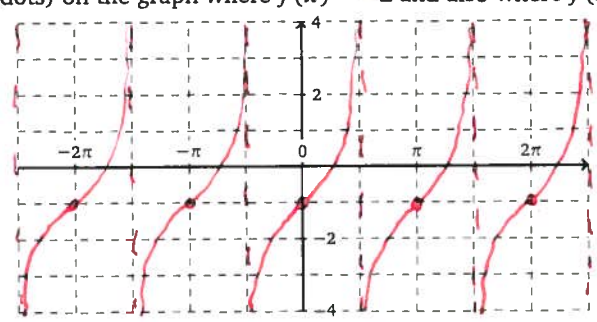
4. *(4 points)*
   (*a*) Write down one of the co-function identities.
   (*b*) Which trigonometric functions are even?

(a) $\cos(\frac{\pi}{2}-t) = \sin t$ $\quad \sec(\frac{\pi}{2}-t) = \csc t$
$\sin(\frac{\pi}{2}-t) = \cos t$ $\quad \csc(\frac{\pi}{2}-t) = \sec t$
+ $\tan(\frac{\pi}{2}-t) = \cot(t)$ $\quad \cot(\frac{\pi}{2}-t) = \tan t$

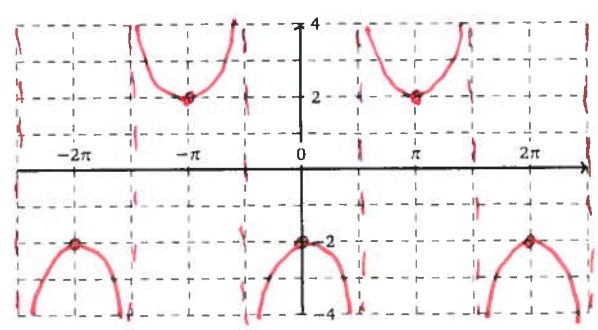
(*a*)
(*b*) $\cos(x)$ + $\sec(x)$

5. *(9 points)* Two functions are given below. On the axes given, draw the asymptotes and the graph of each function, using the entire length of the x-axis. For part (a), draw points (dots) on the graph where $f(x) = -1$. For part (b), draw points (dots) on the graph where $f(x) = -2$ and also where $f(x) = 2$.

+4 (*a*) $f(x) = \tan x - 1$



+5 (*b*) $f(x) = -2\sec x$



6. *(3 points)* While waiting in line to ride the Merry-Go-Round at Lagoon, you estimate that the ride rotates with an angular speed of 4 revolutions per minute. What is the angular speed of the ride in radians per minute?

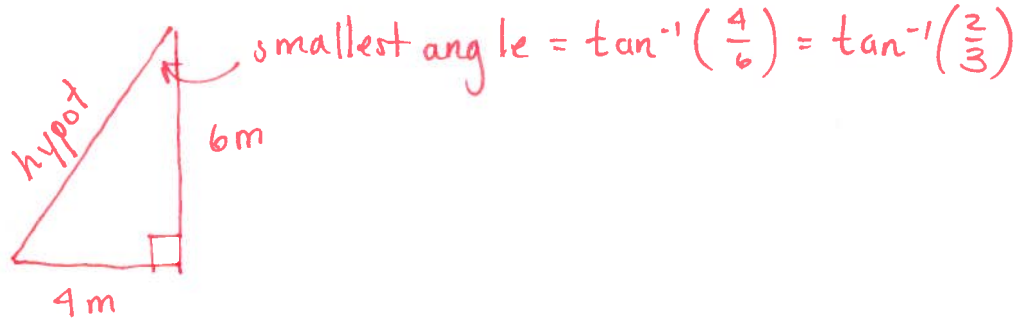
   It is your turn to ride the Merry-Go-Round. If you want to go fast, should you take a seat which is close to the center of the Merry-Go-Round, or a seat which is further away (closer to the edge)? (*Hint*: Consider your linear speed. If it helps, suppose that the Merry-Go-Round has a radius of 20 feet.)

$$4 \frac{\text{rev}}{\text{min}} \cdot 2\pi \frac{\text{rads}}{\text{rev}} = 8\pi \frac{\text{rads}}{\text{min}}$$

$$v = \frac{s}{t} = \frac{r\theta}{t}$$ → Want to sit on a seat with a larger radius r

Angular speed =	$8\pi \frac{\text{rads}}{\text{min}}$
Seat =	Further away

7. *(6 points)* Consider a right triangle with legs measuring 4 meters and 6 meters. What is the measure of the smallest angle of the triangle? What is the measure of the length of the hypotenuse of the triangle? Reminder: Show your work and use correct units. (*Hint*: The angle will be a trigonometric expression; the hypotenuse will be an expression involving a square root.)

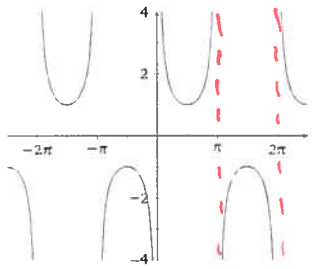


$$\text{hypot} = \sqrt{4^2 + 6^2}$$
$$= \sqrt{16 + 36}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13}$$

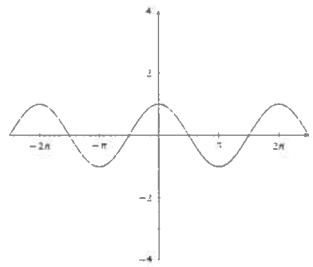
Smallest angle =	$\tan^{-1}\left(\frac{2}{3}\right)$ rads
Length of hypotenuse =	$2\sqrt{13}$ meters

8. *(3 points)* The plots of three trigonometric functions are given below. The functions have **no** transformations. Write the equation corresponding to each graph.

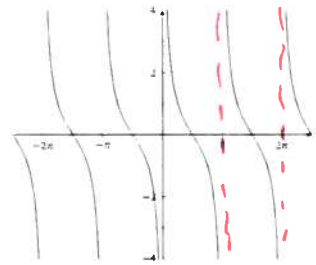
(a)



(b)



(c)



→ period = 2π
has asymptotes
when $\sin x = 0$

(a) $y =$ csc x

(b) $y =$ cos x

(c) $y =$ cot x

9. *(9 points)* Circle each expression which, when simplified, is equal to 1. Show at least one step of your work in the space below each expression.

$\sin x \tan x$ $\tan x = \frac{\sin x}{\cos x}$ $\rightarrow \neq 1$	$\sec^2 x - \tan^2 x$ $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 = \sec^2 x - \tan^2 x$	$\frac{\cos x}{\sec x} = \cos^2 x$ $\neq 1$
$\sin^2 x + \cos^2 x$ Pythagorean thm.	$\sqrt{\sin x \tan x \sec x}$ $\sin x \tan x \sec x = \frac{\sin^2 x}{\cos^2 x}$ $\rightarrow \neq 1$	$\tan x \cot x$ Reciprocal identity $\cot x = \frac{1}{\tan x}$

10. *(4 points)* Expand and simplify $(\sin x + \cos x)^2$.

$(\sin x)^2 + 2(\sin x)(\cos x) + (\cos x)^2$

$(\sin^2 x + \cos^2 x) + 2 \sin x \cos x$

$1 + 2 \sin x \cos x$

— or —

$1 + \sin 2x$

(From later sections in textbook.)

$(\sin x + \cos x)^2 =$ $1 + 2 \sin x \cos x$

11. *(6 points)* Verify the following identity.

$$\frac{\sec^2 x - 1}{\sec^2 x} = \sin^2 x$$

Clearly show your work. (*Hint*: Simplify the left-hand side. Compare the result with the right-hand side.)

$$\frac{\sec^2 x - 1}{\sec^2 x} = \frac{\sec^2 x - 1}{\frac{1}{\cos^2 x}} = (\sec^2 x - 1) \cdot \frac{\cos^2 x}{1}$$

$$= \cancel{\sec^2 x \cdot \cos^2 x} - \cos^2 x$$

$$= 1 - \cos^2 x$$

$$= \sin^2 x \checkmark$$

12. *(3 points)* Factor the following expressions. Clearly show your work.
   (*a*) $\sin^2 x - 5 \sin x \cos x$
   (*b*) $\tan^2 x - 9$

(a) $\sin x \cdot \sin x - \sin x \cdot (5 \cdot \cos x)$

$= \sin x \cdot (\sin x - 5 \cdot \cos x)$

(b) $a^2 - b^2 = (a-b)(a+b)$

$a = \tan x$

$b = 3$

$\rightarrow = (\tan x - 3) \cdot (\tan x + 3)$

(*a*) $\sin x (\sin x - 5 \cos x)$

(*b*) $(\tan x - 3) \cdot (\tan x + 3)$